

# EITM Homework Assignment Day Two Lunch (and Evening)

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*Instructions.* Work on the first two problems first, both of which rely on the following scenario. Work on the final two problems time permitting during lunch, or this evening as part of the lab session.

Suppose we are studying a society where pairs of individuals regularly interact in one-shot games. Suppose that for the two players payoffs are assigned accordingly:

		Player 2	
		<i>C</i>	<i>D</i>
Player 1	<i>C</i>	$2b - c, 2b - c$	$b - c, b$
	<i>D</i>	$b, b - c$	$0, 0$

When  $c > b$ , this is a prisoner's dilemma, and we would expect mutual defection. But in some cases  $c < b$ , and we would expect to see cooperation. Without loss of generality, normalize  $b = 1$ .

1. Assume that in a particular society  $c$  is drawn from a Normal distribution with mean  $\mu$  and standard deviation one; i.e.,  $c \sim \mathcal{N}(\mu, 1)$ . As the researcher you observe  $i = 1, \dots, n$  interactions. You observe whether a particular interaction results in mutual cooperation ( $y_i = 1$ ) or not ( $y_i = 0$ ). Derive the log-likelihood function for this model to estimate the parameter  $\mu$ . Then, write an R program to maximize the log-likelihood and compute the standard error of  $\hat{\mu}$ . Your dataset is:

0, 0, 0, 0, 0, 0, 0, 1, 1, 1

2. Assume that in a different society  $c$  is drawn from a Normal distribution with mean  $\mu$  and standard deviation one; i.e.,  $c \sim \mathcal{N}(\mu, 1)$ . However, a proportion of the society  $\pi$  consists of altruists who cooperate regardless of the value of  $c$  ( $0 \leq \pi \leq 1$ ). As the researcher you observe  $i = 1, \dots, n$  interactions. You observe whether a particular interaction results in one cooperation and one defection ( $y_i = 2$ ), mutual cooperation ( $y_i = 1$ ), or mutual defection ( $y_i = 0$ ). Derive the log-likelihood function for this model to estimate the parameters  $\mu$  and  $\pi$ . Then, write an R program to maximize the log-likelihood and compute the standard errors for  $\hat{\mu}$  and  $\hat{\pi}$ . Your dataset is:

0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 2, 2

3. Degroot and Schervish (2002), p. 363, Exercise 9.
4. For Exercise 3, write an R program to numerically compute the MLE of  $\theta$  for the dataset:

0.09, 0.99, 0.61, 0.70, 0.46, 0.54, 0.18, 0.61, 0.98, 0.85

Also compute the asymptotic standard error for  $\hat{\theta}$ . How do these compare to the analytical solution you found?