

Survival of the Weakest: Why the West Rules[☆]

David K. Levine¹, Salvatore Modica²

Abstract

We study a model of institutions that evolve through conflict. We find that one of three configurations can emerge: an extractive hegemony, a balance of power between extractive societies or a balance of power between inclusive societies. As extractive societies are assumed to have an advantage in head to head confrontations we refer to the latter possibility as the survival of the weakest. Our contention is that the reason the West “rules” was because the invention of the cannon in China made possible the survival of the weakest in Europe, while the arrival of Genghis Khan led to the survival of the strongest in China.

JEL Classification Numbers: D74, C73, D02

Keywords: Game Theory, Evolution, Balance of Power, Conflict, Hegemony, Innovation, Industrial Revolution

[☆]First Version: February 12, 2018. We would like to thank Juan Block, Michele Boldrin, Andrea Ichino and seminar participants at the University of Warwick.

*Corresponding author David K. Levine, 1 Brooking Dr., St. Louis, MO, USA 63130

Email addresses: david@dklevine.com (David K. Levine), salvatore.modica@unipa.it (Salvatore Modica)

¹Department of Economics and RSCAS European University Institute and Department of Economics WUSTL

²Università di Palermo, Italy

1. Introduction

We study a model of institutions that evolve through conflict. We find that one of three configurations can emerge: an extractive hegemony, a balance of power between extractive societies or a balance of power between inclusive societies. As extractive societies are assumed to have an advantage in head to head confrontations we refer to the latter possibility as the survival of the weakest. Our contention is that the reason the West “rules” was because the invention of the cannon in China made possible the survival of the weakest in Europe, while the arrival of Genghis Khan led to the survival of the strongest in China.

There are as many theories of “why the West rules,” which is to say, why the industrial revolution took place in the West rather than, say, China, as there are historians and economists.³ One element on which there seems to be agreement⁴ is that competition between relatively inclusive institutions such as those in Western Europe are more likely to generate innovation - and an industrial revolution - than the relatively extractive hegemonies found in China.⁵ Accepting this basic conclusion we must ask: why was there competition between relatively inclusive institutions in Europe while in China we find an extractive hegemony? Why did India - made up of competing societies not an extractive hegemony - generate relatively little innovation?

Some, such as Diamond (1998), have argued that the geography of Europe is more favorable to competing societies than China. This not only leaves unanswered the question of India, but a careful examination of a map by Hoffman (2013) shows that the premise is not true.⁶ We think that answers to questions about hegemony and institutions must be found in models of institutional evolution. We observe a wide array of different institutions both across space and time: political systems in particular range from quite democratic to relatively autocratic - inclusive or extractive or inclusive in the terminology of Acemoglu and Robinson (2012). There are competing societies in some times and places and hegemonies in others. We observe that there is a connection: historically people and institutions have more often spread through invasion and conflict than through peaceful change.⁷ In addition, significant institutional change has most often arisen in the aftermath of the disruption caused by warfare and other conflicts between societies. Consequently we are led to a theory of institutional change that arises from conflict, as in Rosenthal and Wong (2011), and not, as in Bisin and Verdier (2001) or Greif and Tabellini (2010), from internal evolution within a single society.

In an evolutionary model it seems that extractive institutions generally levy higher taxes and

³The astute reader will note a reference to a recent such book Morris (2010) in our title. McNeil (1963) and Cipolla (1965) are some earlier contributions to what is now an extensive literature.

⁴See, for example, Landes (2003), Lin (1995) and Liu and Liu (2007).

⁵Why the character of the industrial revolution was different than earlier episodes of innovation we do not address - for discussion of this issue see Mokyr (2002) and Dutta et al (2018).

⁶Hoffman (2013) argues that a key difference between Europe and China was the role of the Catholic Church in Europe - it plays a role in his theory similar to the outsiders in ours.

⁷This was apparently true even in the earliest of times. Bowles and Choi (2013) argue that farming was initially an inferior technology to foraging and became widespread not because it was eagerly adopted by imitators, but rather because farmers had “formidable military technology” that enabled them to successfully encroach on foragers.

have larger armies: if evolution is driven by conflict why do they not predominate over “weaker” inclusive institutions? In our earlier work Levine and Modica (2013) we suggested that the answer was to be found in the presence of outsiders: while models of evolutionary conflict generate a tendency towards extractive hegemonies, interfering outsiders such as English or Central Asians made it difficult to form extractive hegemonies in Europe and India. Unfortunately that theory left unanswered the question of what happens when hegemonies do not form and why the record of innovation in India is so much different than that in Europe.

Here we re-examine the evolutionary conflict model. We simplify the model by omitting the details of conflict found in Levine and Modica (2013) and strengthen it along the lines suggested in Levine and Modica (2017) by adding a “home field advantage” for a society defending its own land. In this simple model two societies with two possible institutions - inclusive or extractive - compete through conflict. Each society has two groups - masses and elites - and conflicts occur based on the incentives of these groups.

Our basic finding is that there are only three possible stochastically stable outcomes: an extractive hegemony, a balance of power between two extractive societies, and a balance of power between two inclusive societies. The last we refer to as “survival of the weakest.” In our model in a head to head contest between an extractive and inclusive society the extractive society is more likely to prevail. Never-the-less we show that competition between inclusive societies may persist in the long-run. The circumstances that favor this are strong outsiders - as in Levine and Modica (2013) - and military technology in which strong defensive forces are important. We argue that an important difference between early medieval military technology and later military technology is that the advent of the cannon made it impossible to defend with little effort behind a secure wall and made defense a far more demanding endeavor.

After we develop the basic model and prove our main theorem characterizing stochastically stable states we apply it to see how well our two variables - outsiders and military technology - explain the history of hegemony and institutions (and by implication innovation). We argue that the West “won” less because of events in the West and more because of two developments in China: the invention of the cannon and the arrival of Genghis Khan.

2. The Model

Two societies (or countries) with the same technology contend over land and the people, the physical capital, and the other resources that reside there. There are two units of land, one for each society. There are two possible configurations: a *balance* of power in which each society occupies its own unit of land and a *hegemony* in which one society, the *occupier*, occupies both units of land and the other society is referred to as the *occupied*.

There are two groups in each society: the *masses* and the *elite*. There are two types of institutions, inclusive institutions w and extractive institutions s . Roughly speaking with inclusive institutions the masses have the upper hand, while with extractive institutions the elite have the upper hand. Depending on circumstances either society may have either type of institution. There

are five possible states: $z \in Z = \{w, s, ww, sw, ss\}$. The first two correspond to a hegemony in which the occupier has inclusive and extractive institutions respectively and the remaining three correspond to a balance of power in which both have inclusive, one has extractive, the other inclusive, and in which both have extractive institutions.

Conflict between societies takes place over time $t = 1, 2, \dots$. At the beginning of period t there is a status quo given by the state from the previous period z_{t-1} . A game between the two groups in the two societies is played and the outcome determines the state z_t in the current period. The particular game depends upon the status quo z_{t-1} and a iid random shock. It takes place in two stages. In the first stage only one of the four groups is *active* and may decide to initiate a conflict to achieve a particular *goal*. The decision is based on a stochastic utility shock. If the active group is part of an occupied society the conflict is a *rebellion* to liberate their land and the goal is to install particular institutions there. If the active group is part of a balance of power the conflict is to *attack* the other society and the goal is to occupy their land. If the active group chooses not to initiate a conflict the status quo remains unchanged and $z_t = z_{t-1}$. If the active group initiates a conflict a second stage simultaneous move game is played. The active group initiating the conflict is designated as the *aggressor* and one group from the opposing society is the *defender*. Each simultaneously decides the level of effort to devote to the conflict and these effort levels stochastically determine the new state. All of the groups are myopic in the sense that they care only about the consequences of their actions in the current period. All random events are iid conditional on the state: they do not otherwise depend on history.

Outsiders

In addition to the four decision making groups of *insiders* there are a number of *outsiders* whose strength relative to the insiders is denoted by $\eta > 0$. These outsiders do not make decisions but help determine in an important way the environment in which conflict takes place. They represent the global picture of the context in which the two societies contend: they represent societies and people outside of the model who are protected from the insiders. They may be protected by asymmetrical geographical barriers or by superior force. The represent forces that may be able to “get at the insiders” but cannot easily be “got at by the insiders.” Both geography and technology matter: the English channel was not a barrier to continental invasion given English and Roman technology in Julius Caesar’s time post. After 1400 naval technology and standing navies favored strongly the short coastline of England over the long coastline of continental Europe so that England could interfere easily in the continent but not so easily be invaded by the continent. Hence in the centuries long conflict between France and Austria/Germany the country of England was effectively an outsider.

Our basic hypothesis is that outsiders are disruptive of hegemony but supportive of a balance of power. As discussed in detail in Levine and Modica (2017) this is in broad accordance with historical facts. The most notable example is the role of England in maintaining a balance of power on the continent, which is well documented and is notorious for its complete cynicism. From the rise of Spain following the discovery of America in 1492 through Brexit in 2016 British foreign policy

has largely been aimed at preventing a hegemony in continental Europe. Many books (see for example Sheehan (1996)) have been written on the topic and few discussions of European history fail to observe the remarkable fact that Britain consistently changed sides in conflicts to support the weak. Most dramatic, perhaps, is the shift to an alliance with France in 1904 in the face of the German threat - this after nearly 1000 years of historical enmity against the French culminating in what many consider to be the true first world war: the Napoleonic wars. Note that until the advent of the European Union and the fall of the Iron Curtain this English policy was quite successful.

Outsiders can be disruptive to hegemony in a variety of ways. First, they may provide a refuge in which governments-in-exile can regroup and in which rebel forces indeed may accumulate. For example, in 1994 the Hutu hegemony in Rwanda along with the genocide it incited came to an end when rebel Tutsi forces invaded from Uganda where they had safe haven. Second, outsiders may provide direct support in the form of weapons or even troops. For example, in the final battle of the US revolutionary war in which the British were decisively defeated the rebel force consisted of 11,000 US soldiers - together with 8,000 French soldiers and crucially the support of a French naval fleet.

We now describe in greater detail the game and payoffs: the model is simple and stylized. Subsequently we analyze the robustness of the main results to departures from this basic model.

The Initiation of Conflict

In a balance of power each society has an equal chance of being active. In a society with inclusive institutions the active group is the masses. In a society with extractive institutions the active group is the elite. The goal is to occupy the land belonging to the other society.

In an inclusive hegemony the active group is the occupied masses. In an extractive hegemony the active group is the occupied elite. There are two possible goals. With positive probability less than one the goal is to revolt and install inclusive institutions and with the remaining probability the goal is to revolt and install extractive institutions. The probability may depend upon current institutions: it may be that when the masses are calling the shots it is more likely that the goal will be revolt to inclusive institutions than when the elite are calling the shots. Notice that we assume an element of culture inherited from the occupying power - groups in the occupied territory are assumed to operate under the institutions imposed on them by the occupier. In a society run by generals it is generals who are most likely to rebel. In a society where the support of the masses is essential a revolt is only possible with their support. We refer here, in particular, to the fact that in the process of decolonization the support of the masses played an important role despite the fact that in precolonial times they had little voice. Notice, however, that a revolt may adopt either type of institution depending upon the goal. It is possible, for example, that the masses would agree to extractive domestic institutions in return for liberation from foreign domination by inclusive institutions.

Once the active group and goal are determined an iid random utility shock \tilde{u} occurs: this is standard for a random discrete choice model. The active group then decides whether or not to initiate conflict - to attack or revolt. If the active group decides not to initiate a conflict the game

ends and the state remains unchanged. In this case $z_t = z_{t-1}$ and the utility of all groups is that in the status quo. If the active group decides to initiate conflict the utility of the active group is increased by \tilde{u} , the current state z_t is randomly determined through conflict resolution, and the utility of all groups is determined by the current state minus the costs of conflict.

As conflict - at least in the sense of an all-out revolt or attempt to occupy a foreign nation - is rare, we assume that the utility shock is with high probability negative. If \tilde{u} is very negative the active group will not choose to initiate a conflict, so it is only the upper tail of this random variable that matters. We assume this has an exponential form given by three parameters $U > 1$, $0 < P < 1$ and $\sigma > 0$ so that if $v \geq -U$ then $\Pr(\tilde{u} \geq v) = Pe^{-\sigma(v+U)}$. With probability $1 - P$ the shock is smaller than $-U$ and no conflict is initiated. The parameter σ is a scale parameter for the utility shock distribution. If it is large the probability of a shock much bigger than $-U$ is very small. We will be interested in the case in which σ is large.

Conflict Resolution

If conflict takes place the active group - now called the *aggressor* - determines the level of effort $1 \geq x_a \geq 0$ to devote to the conflict. In a balance of power the defender is the masses if the society under attack has inclusive institutions and the elite if the society under attack has extractive institutions. In an inclusive hegemony the defender is the occupier masses; in an extractive hegemony the defender is the occupier elite. The defender determines a level of effort $1 \geq x_d \geq 0$ to devote to the conflict. Each contestant group $i \in a, d$ faces a quadratic cost of effort provision $C(x_i) = (\gamma/2)x_i^2$ where $\gamma \geq 1$. Note that the two groups who are neither aggressor nor defender do not bear any cost of conflict - although if they did it would not matter since they have no decision making power.

Let $\zeta \in \{h, b\}$ be an indicator of whether the state is hegemonic or a balance of power. The probability the aggression succeeds depend on the resources committed by the contestant groups and is given by a conflict resolution function⁸ $\pi(x_a, x_d) = \Pi_\zeta(\eta) + \alpha(x_a - [(1 - \varphi)x_d + \varphi])$ where $0 \leq \varphi \leq 1$, $\Pi_\zeta(\eta) > 0$ is continuous with $\Pi_h(\eta)$ strictly increasing in η , $\Pi_b(\eta)$ strictly decreasing in η and $\alpha > 0$. We assume, moreover, that $\max\{\Pi_h(\infty), \Pi_b(0)\} + \alpha < 1$ and $\min\{\Pi_h(0), \Pi_b(\infty)\} - \alpha > 0$ so that regardless of the choices of effort and value of φ the probability of success is positive but not certain. Finally, we assume that $\Pi_h(\infty) > \Pi_b(\infty)$ and $\Pi_b(0) > \Pi_h(0) + \alpha$.

Notice that if aggressor and defender employ equal effort, $x_a = x_d = x$, then $\pi(x, x) = \Pi_\zeta(\eta) - \alpha\varphi(1 - x)$. This decreases in φ , and as x goes up it increases from $\pi(0, 0) = \Pi_\zeta(\eta) - \alpha\varphi$ to $\pi(1, 1) = \Pi_\zeta(\eta)$. If $\Pi_\zeta(\eta)$ is small this represents an intrinsic advantage of being a defender. In a hegemony the defending hegemon has the advantage of greater resources and the disadvantage of defending on foreign soil. In a balance of power the defender does not have the resources of a hegemon but does have the advantage of defending home turf. How great these advantages or disadvantages are depends on the strength of outsiders: our assumption dependence of $\Pi_\zeta(\eta)$ on

⁸See Hirshleifer (2001) and Hausken (2005) for a broader discussion of the use of conflict resolution functions.

η is consistent with our view of that outsiders help rebels chances of success but hurt those of an aggressor in a balance of power. The assumptions on the limits of Π_ζ say that outsiders are potentially important in the sense that (given efforts) if they are strong enough then rebels have a better chance of success than balance of power aggressors and if they are weak then balance of power aggressors have a better chance of success than rebels. Notice the implication that there is a unique value η^* such that $\Pi_h(\eta^*) = \Pi_b(\eta^*)$.

The parameter α measures the sensitivity of the outcome to the differential effort of the two combatants. We have assumed that this is not too large. The parameter φ measures the sensitivity of the outcome to defensive effort. The coefficient on x_d is $(1 - \varphi)x_d + \varphi$ a weighted average of the defensive effort and 1. The interpretation is that 1 measures the value of fixed fortifications. If these are strong the effort of defenders should not matter much so φ should be large. If siege technology is effective - for example cannons can knock down defensive walls - then φ should be small. Hence we interpret φ as the effectiveness of fortifications.

If the aggression fails the status quo remains unchanged, $z_t = z_{t-1}$ and the utility of all groups is that in the status quo less the effort and plus the utility shock. If the status quo is a balance of power and the aggression succeeds the new state is a hegemony with the institutions of the aggressor. If the status quo is a hegemony and the aggression succeeds the new state is a balance of power in which the defender institutions are unchanged and the aggressor institutions are those determined by its goal. In all success cases the utility of all groups is that of the new state less the effort and plus the utility shock.

Incentives: Transfers and Utility

In addition to the random utility shock and conflict costs the utility of groups is determined by the current state z_t . There are two possible tax levels representing a transfer from the masses to the elite on each unit of land. High taxes are normalized to 1 and low taxes are $\tau > 0$. In a hegemony the elite receive the taxes from both units of land; in a balance of power the elite receive the taxes from their own land only. Taxes are low in an inclusive state either in a balance of power or for the occupier of a hegemony. Taxes are high if institutions are extractive and for the occupied regardless of institutions. We can summarize the transfer utility of the masses and elites respectively depending on institutions and configuration in the following table:

	s	w
balance	$-1, 1$	$-\tau, \tau$
occupier	$-1, 2$	$-\tau, 1 + \tau$
occupied	$-1, 0$	$-1, 0$

Under extractive institutions the masses always pay high taxes so receive a utility of -1 . Occupied masses also pay high taxes and receive a utility of -1 . Domestic masses with inclusive institutions pay low taxes so have a utility of $-\tau$. In a balance of power the elites collect domestic taxes. In a hegemony the occupier elites collect all the taxes and the occupied elites get none.

While it is convenient to describe the benefits and costs of occupation in terms of taxes, the key features are that there is a preference against foreign rule by both masses and elites and that occupier elites benefit from occupation at the expense of the occupied elites. For example, it may be that masses dislike foreign rule because of issues of language or distaste for foreigners. It may be that occupier elites enjoy the status that comes with ruling over a larger territory. We take as a relatively neutral assumption that masses are indifferent between being exploited by foreigners and by domestic elites and that occupier elites receive the same benefits from occupied territories as they do from extractive institutions in their domestic territory and do so at the expense of the occupied elites.

Equilibrium

An equilibrium is the stochastic process in which a Nash equilibrium occurs within each period. We will show that this equilibrium is unique, so depends only on the state $z \in Z = \{w, s, ww, sw, ss\}$. Likewise the probability of current state conditional on the within period equilibrium depends only on the previous state. Hence an equilibrium is a Markov process on the state space Z .

Notice that there is a positive probability of remaining in place and a positive probability of each of the eight feasible transitions. Hence the process is aperiodic and ergodic, which in turn implies that there is a unique ergodic probability distribution μ_σ over the state space describing how frequently the different states are visited. From Young (1993) we also know that as $\sigma \rightarrow \infty$ the ergodic distributions μ_σ have a unique limit μ . Those states that have positive probability in the limit distribution μ are called *stochastically stable*: they represent the state or states which are observed “most of the time” when σ is large. As we are interested in the case where σ is large - that is serious conflict is infrequent - we will characterize the stochastically stable states.

3. Stochastic Stability

We now state the main result of the paper which characterizes persistent institutional configurations. As to the “survival of the weakest”, as anticipated in the introduction it emerges as a long run possibility with strong outsiders and ineffective fortifications. The role of the φ parameter is more subtle than it may appear: it is true that as it becomes lower more defense effort is needed, but it is equally true that the probability of success with equal forces increases, so the result is not due to the mechanics of conflict directly. Rather, technology affects the economic incentives to subvert the status quo order in the conflict subgame, and these turn out to be weaker with low φ .

Theorem 1 (Main Theorem). *For generic values of the parameters there is a unique stochastically stable state. Only s, ss, ww can be stochastically stable. There exists a $0 < \tau^* < 1$ and a strictly decreasing function $0 < \varphi_\tau < 1$ such that*

1. *if $\tau > \tau^*$ or $\varphi > \varphi_\tau$ then ww is not stochastically stable with s stochastically stable for $\eta < \eta^*$ and ss stochastically stable for $\eta > \eta^*$*

2. if $\tau < \tau^*$ and $\varphi < \varphi_\tau$ then *ss* is not stochastically stable and there is a positive continuous strictly decreasing function $\eta(\varphi) \leq \eta^*$ with *s* stochastically stable for $\eta < \eta(\varphi)$ and *w* stochastically stable for $\eta > \eta(\varphi)$.

To parse this result, consider first the case $\tau > \tau^*$, which is to say inclusive institutions do not offer such a great advantage over extractive institutions. In this case we find that inclusive institutions are never stochastically stable. Roughly: the masses are unwilling to make much effort to defend inclusive institutions that are not all that inclusive. We do not find this fact terribly interesting: there are institutions of varying degrees of inclusiveness - and our focus naturally is on whether sufficiently inclusive institutions may survive. Never-the-less this result is interesting: it implies that we will not often see “somewhat inclusive” institutions, only extractive or “strongly inclusive” institutions. There are two ways in which we can make this observation precise. We could analyze a setting with three types of institutions - that is beyond the scope of this paper. We might also consider that institutions evolve within societies. We do not propose to explicitly analyze such a model - we refer the reader to the excellent survey of Bisin and Verdier (2005) for a review of models of internal institutional change. Rather we consider the claim made by Olson (1982) and others since that inclusive institutions tend to become less so over time. In this model if we start with parameters such that a balance of power between inclusive institutions is stochastically stable (low τ) and over time τ gradually rises we will initially see a gradual deterioration of living standards for the masses - followed by a rather catastrophic collapse to extractive institutions once the threshold τ^* is crossed. This is reminiscent of Hayek (1944)’s road to serfdom. We should say, however, that evidence for such a scenario is poor. On the one hand, although current politics are turning against globalization, proposals by populist parties are in fact rather modest and scarcely propose turning the clock back to 1982. On the other hand it seems that events in China must be described as the “road away from serfdom” as current Chinese institutions hardly rival in extractiveness those in the time of the cultural revolution or great leap forward.

For the remainder of the paper we are going to focus on the case $\tau < \tau^*$, that is “strongly inclusive” institutions and ask how they fare against extractive institutions. A careful reading of theorem 1 shows that if $\eta < \min_\varphi \eta(\varphi)$ then only extractive hegemony is stochastically stable. If there are sufficiently few outsiders then we should generally observe extractive hegemonies - and this is true regardless of military technology.

For stronger outsiders, that is, larger values of η , we will see a balance of power - but military technology determines which type. When φ is large (and specifically $\varphi > \varphi_\tau$) then an extractive balance of power is stochastically stable, while if φ is small then an inclusive balance of power is stochastically stable. Large φ - effective fortifications - favors extractive institutions, while small φ - good siege technology - favors inclusive technology. To anticipate: the invention of gunpowder led to a great reduction in the effectiveness of fortifications. This should have led to inclusive institutions - as it did in Europe and not generally speaking in China or India. We will examine this history in greater detail below.

Before getting to history the remainder of this section is devoted to proving Theorem 1. As

usual we do this by backwards induction - analyzing the final conflict subgame first before analyzing the overall game.

The Conflict Subgame

The starting point is to find the equilibrium \hat{x}_a, \hat{x}_d of the subgame in which conflict takes place. We will show that this equilibrium is unique and does not depend on the state or strength of outsiders. It does depend on y_a the transfer benefit to the prospective aggressor from the change in state and on y_d the loss to prospective defender from the change in state. Note that these values lie between 0 and 1. Define the *partial utility gain* to the aggressor as the part of the equilibrium gain that is independent of the state and strength of outsiders $u(y_a, y_d, \varphi) = \alpha(\hat{x}_a - [(1 - \varphi)\hat{x}_d + \varphi])y_a - (\gamma/2)\hat{x}_a^2$. The *overall utility gain* of the aggressor can then be written as $\Pi_\zeta(\eta)y_a + u(y_a, y_d, \varphi)$. It is always less or equal than 1. We report the relevant facts proven in the Appendix:

Theorem 2. *The conflict subgame has a unique Nash equilibrium independent of the state and strength of outsiders. The total utility gain to the aggressor $\Pi_\zeta(\eta)y_a + u(y_a, y_d, \varphi)$ is non-negative, less than or equal to one and strictly increasing in y_a . The partial utility gain $u(y_a, y_d, \varphi)$ is strictly increasing in φ , decreasing in y_d , and satisfies $u(0, y_d, \varphi) = 0$. There is a $0 < \tau^* < 1$ and a strictly decreasing function $0 < \varphi_\tau < 1$ such that the function $v(\varphi) \equiv u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi) - u(1 - \tau, 0, \varphi)$ satisfies $|v(\varphi)| < \alpha$ with $v(\varphi) < 0$ for $\tau < \tau^*$ and $\varphi < \varphi_\tau$ and $v(\varphi) > 0$ otherwise.*

Proof of the Main Theorem

Proof. [of Theorem 1] Under our assumptions with high probability the state remains unchanged: we are interested in the probabilities of transitions that change the state. The *ex ante* probability of a successful implementation of the particular goal has the form

$$Q \cdot \Pr(\Pi_\zeta(\eta)y_a + u(y_a, y_d, \varphi) + \tilde{u} \geq 0) = Qe^{-\sigma[U - \Pi_\zeta(\eta)y_a - u(y_a, y_d, \varphi)]}$$

where Q takes account of the probability that the group is in fact active and that a particular goal is on the table. The key point is that in our model Q is independent of σ and bounded away from 0 and 1. This enables us to analyze stochastic stability using the standard notion of resistance (see, for example, Young (1993) or Kandori, Mailath and Rob (1993)). Here we take $\epsilon = e^{-\sigma}$ so that as $\sigma \rightarrow \infty$ then $\epsilon \rightarrow 0$. The resistance then is the derivative of the logarithm of the probability with respect to the logarithm of ϵ . Hence resistance is given simply by $U - \Pi_\zeta(\eta)y_a - u(y_a, y_d, \varphi)$.

Following this preamble we may easily compute the resistances. Let $r(z \rightarrow z')$ denote the resistance of the transition from z to $z' \neq z$. Observe that the Markov process has a birth-death like structure: each state can at most move only to one of two adjacent states. From ss the only non-trivial transition is to s . From s it is possible only to transition back to ss or forward to sw , and so forth. In the table below we lay out the resistances of the non-trivial feasible transitions. We indicate first the transition then the benefit from the transition to the masses and elites respectively. This is divided into two columns for occupier or occupied; if the initial state is a hegemony these are

the hegemony's four groups; in case the initial state is a balance of power occupier versus occupied refers to the situation after transition - for example in the transition $sw \rightarrow s$ the inclusive society's groups end occupied and extractive society's groups become occupiers; the s -elite have $y_a = 1$ while the w -elite have $y_d = \tau$. The asterisks denote which group is making the decision about attack or paying for defense. Note that the defender never gains from a transition and consequently the negative values in the table always correspond to the defender. The value y_d is the negative of the defender "benefit" reported in the table: it represents the loss avoided when the defenders prevent the transition from taking place. The final column reports the resistance.

transition	occupier	occupied	$r(z \rightarrow z')$
$ss \rightarrow s$	$0, 1^*$	$0, -1^*$	$U - \Pi_b(\eta) - u(1, 1, \varphi)$
$s \rightarrow ss$	$0, -1^*$	$0, 1^*$	$U - \Pi_h(\eta) - u(1, 1, \varphi)$
$s \rightarrow sw$	$0, -1^*$	$1 - \tau, \tau^*$	$U - \Pi_h(\eta)\tau - u(\tau, 1, \varphi)$
$sw \rightarrow s$	$0, 1^*$	$-(1 - \tau)^*, -\tau$	$U - \Pi_b(\eta) - u(1, 1 - \tau, \varphi)$
$sw \rightarrow w$	$0^*, 1$	$0, -1^*$	$U - u(0, 1, \varphi)$
$w \rightarrow sw$	$0^*, -1$	$0^*, 1$	$U - u(0, 0, \varphi)$
$w \rightarrow ww$	$0^*, -1$	$1 - \tau^*, \tau$	$U - \Pi_h(\eta)(1 - \tau) - u(1 - \tau, 0, \varphi)$
$ww \rightarrow w$	$0^*, 1$	$-(1 - \tau)^*, -\tau$	$U - u(0, 1 - \tau, \varphi)$

The attribute of a state that determines the relative time the process spends in it is the *modified radius*. We denote by $R(z)$ the modified radius of the state z . The critical fact is that the stochastically stable states are exactly those with the greatest modified radius. The general definitions are given in Levine and Modica (2016), section 6.3, we sketch here the concept to apply it to the present setting. We say that a collection of states form a *circuit* if any two of them are connected by a least resistance path. The states can be partitioned into circuits, and then one defines circuits of circuits - 2nd-order circuits we may say - by taking as modified resistance the incremental resistance needed to move from one to the other over that needed to move within the circuit. It is possible to define even higher order circuits: the highest order circuit in this model is the 3rd order circuit consisting of all the states.

We start by determining the structure of circuits. We observe that $r(s \rightarrow sw) = U - \Pi_h(\eta)\tau - u(\tau, 1, \varphi) > U - \Pi_h(\eta) - u(1, 1, \varphi) = r(s \rightarrow ss)$ by the monotonicity property of $u(y_a, y_d, \varphi)$. This implies that ss and s form a circuit. Also by monotonicity we have $r(w \rightarrow ww) = U - \Pi_h(\eta)(1 - \tau) - u(1 - \tau, 0, \varphi) < U - u(0, 0, \varphi) = r(w \rightarrow sw)$ so that ww and w form a circuit. The only remaining question is at the next level which circuit sw joins. It will join the circuit to which it has the least resistance of reaching. By monotonicity $r(sw \rightarrow s) = U - \Pi_b(\eta) - u(1, 1 - \tau, \varphi) < U - u(0, 1, \varphi) = r(sw \rightarrow w)$ so the resistance of joining the $ss \leftrightarrow s$ circuit is less than joining the $w \leftrightarrow ww$ circuit. We may summarize the situation of circuits by the following diagram: $[(ss \leftrightarrow s) \leftrightarrow sw] \leftrightarrow (w \leftrightarrow ww)$, where round brackets mean "in a circuit with" and square brackets mean "in a 2nd order circuit with" and the entirely forming a 3rd order circuit.

In general we compute modified radii by moving between circuits adding in the incremental cost of moving up to the next level until the highest level circuit is joined.⁹ This can be complicated since there may be many ways of moving between states, but in this model because of the birth-death like structure matters are simplified since there is exactly one way to get from one state to another. Start with s . To get to the highest level we must move to sw then to the other circuit at w . The radius of s is $r(s \rightarrow ss)$. To this we add the incremental resistance of moving to sw which is $r(s \rightarrow sw) - r(s \rightarrow ss)$ and the incremental resistance of moving to w which is $r(sw \rightarrow w) - r(sw \rightarrow s)$. Denoting the modified radius by $R(z)$ we then have

$$\begin{aligned} R(s) &= r(s \rightarrow ss) + r(s \rightarrow sw) - r(s \rightarrow ss) + r(sw \rightarrow w) - r(sw \rightarrow s) \\ &= r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s) \end{aligned}$$

Notice how the backward looking part $r(s \rightarrow ss)$ cancels out. This is a general property in a birth-death like model in which movement is one step is in a single dimension.

We may apply this idea to all five states to compute the modified radius. The second column shows how the modified radius depends upon the resistances, the third how they depend on the model parameters. In that computation we make use of the fact that $u(0, y_d, \varphi) = 0$.

state	R	R
ss	$r(ss \rightarrow s) - r(s \rightarrow ss) + r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U + \Pi_h(\eta)(1 - \tau) + [u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi)]$
s	$r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U + \Pi_b(\eta) - \Pi_h(\eta)\tau + [u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi)]$
sw	$r(sw \rightarrow w)$	U
w	$r(w \rightarrow sw)$	U
ww	$r(ww \rightarrow w) - r(w \rightarrow ww) + r(w \rightarrow sw)$	$U + \Pi_h(\eta)(1 - \tau) + u(1 - \tau, 0, \varphi)$

Since $R(ww) > U$ we immediately see that neither sw nor w is stochastically stable. We compare $R(ss) - R(s) = \Pi_h(\eta) - \Pi_b(\eta)$ and $R(ss) - R(ww) = u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi) - u(1 - \tau, 0, \varphi) = v(\varphi)$. We see that between ss and s stochastic stability is determined entirely by whether $\eta > \eta^*$ or $\eta < \eta^*$. Between ss and ww we see that stochastic stability is determined entirely by $v(\varphi)$. Putting this together with Theorem 2 the only fact remaining to prove in the final case for η sufficiently small s is in fact stochastically stable, that is, $R(s) = U + \Pi_b(\eta) - \Pi_h(\eta) + \Pi_h(\eta)(1 - \tau) + [u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi)] > U + \Pi_h(\eta)(1 - \tau) + u(1 - \tau, 0, \varphi) = R(ww)$. This condition simplifies to $\Pi_b(0) - \Pi_h(0) + v(\varphi) > 0$ which is true since $|v(\varphi)| < \alpha$ and we assumed that $\Pi_b(0) - \Pi_h(0) > \alpha$. \square

Survival of the Weakest and Home Field Advantage

Using the tool of resistance introduced in the proof of theorem 1 we are now in a position to explain the notion “survival of the weakest.” Recall that $r(z \rightarrow z')$ denotes the resistance of the transition from z to z' , a measure of the speed of convergence to zero of the probability of the transition - roughly speaking higher resistance means the transition is more difficult. Consider a head to head contest between an extractive and an inclusive society, that is, the state sw . Which

⁹This is similar to the computation of Ellison (2000)'s co-radius.

side is more likely to prevail? The answer is the one with less resistance: if $r(sw \rightarrow s)$ is lower than $r(sw \rightarrow w)$ this means that the extractive society is (much) more likely to prevail over the inclusive society than the other way around. That this is the case follows from the simple calculation $r(sw \rightarrow w) - r(sw \rightarrow s) = \Pi_b(\eta) + u(1, 1 - \tau, \varphi) > 0$. Hence we are justified in referring to an extractive society as “strong” and an inclusive society as “weak.”

We point out another interesting feature of this model: using the fact that $u(0, y_d, \varphi) = 0$ we have $r(ww \rightarrow w) - r(ss \rightarrow s) = \Pi_b(\eta) + u(1, 1, \varphi) > 0$. This says the strong perform well on foreign ground while the weak do not - that is in attacking a strong power the strong do better than the weak do in attacking a weak power. It also embodies the idea of “home field advantage” particularly for the weak. The idea that defending home turf is an advantage is an important feature of this model that differs from our earlier work: we briefly indicate how this model differs from those earlier models. Levine and Modica (2013) and the example of China in Levine and Modica (2016) use a common framework in which there are many units of land and the weaker society always has zero resistance. Here we have simplified by having only a single unit of land belonging to each society - but the same point as we make here could be made in the many units of land case with a border: the weaker power can still have zero resistance - but which power is weaker changes when the border is crossed due to the disadvantage of fighting on enemy territory - an effect that we believe is important but is absent in our earlier work. Here - unlike that earlier work - resistance is endogenous brought about by the decisions of groups and not an exogenous “state power” similar to the notion of “state capacity” used by Besley and Persson (2010). In subsequent work Levine and Modica (2017), also using the many units of land framework, but restricting attention to a single type of society, the intervention of outsiders acted like a border. For the type of conflict studied in that paper where modern societies both have claims to the same land that assumption still seems to us appropriate, but for the situation considered here of evolution between societies which may conquer one another the notion of a border seems most appropriate.

The idea that the strong perform well whether they are fighting “at home” or “away” while the weak perform well only when they are fighting “at home” arises in this model because the masses have little reason to attack but much reason to defend. The fact that the strong do well both at home and away while the weak only do well at home is common phenomenon well known to sports fans. For example, we gathered data from NBA basketball ¹⁰ and found that when weak teams play each other the visiting team won about half as often as when strong teams play each other.

Robustness

While the basic model is highly stylized the results in theorem 2 are robust to many details. We discuss several extensions here.

With respect to the specific linear/quadratic conflict resolution function we observe that theorem

¹⁰Using data for the 2016 NBA season from http://www.espn.com/nba/team/schedule/_/name/gs/year/2017 we computed the records of the three best teams in the Western Conference against each other and the same for the three worst teams.

2 relies only on the conclusion of theorem 2: any conflict technology that results in a $u(y_a, y_d, \varphi)$ function satisfying the qualitative properties of that theorem yield our main theorem, theorem 1. Certainly small perturbations from the linear quadratic model will do so. In fact these conclusions are relatively robust. Consider the more general conflict resolution function $\pi(x_a, x_d) = \Pi_\zeta(\eta) + \alpha\pi_0(x_a, x_b, \varphi)$ where π_0 is strictly increasing in x_a and decreasing in x_b , strictly so for $\varphi < 1$ and independent of x_b for $\varphi = 1$ together with convex effort cost $c(x_a)$. If, as in the base model, the outcome of π_0 is uncertain enough and $c(x_a)$ is convex enough existing results (see Herrera, Morelli and Nunnari (2015)) imply the existence of a unique interior pure strategy equilibrium. Defining $u(y_a, y_d, \varphi) = \alpha\pi_0(\hat{x}_a, \hat{x}_b)y_a - c(\hat{x}_a)$ then certainly $\Pi_\zeta(\eta)y_a + u(y_a, y_d, \varphi)$ is non-negative, less than or equal to one and under mild conditions the intuitive conditions that it is strictly increasing in y_a and that $u(y_a, y_d, \varphi)$ is strictly decreasing in y_d , and satisfies $u(0, y_d, \varphi) = 0$ hold. The key qualitative features of the model require also that $v(\varphi) = u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi) - u(1 - \tau, 0, \varphi)$ is negative for small τ and φ and positive for large φ . Consider first that for $\tau = 0$ and $\varphi < 1$ we have $v(\varphi) \equiv u(1, 1, \varphi) - u(1, 0, \varphi) < 0$ by monotonicity, so indeed $v(\varphi)$ is negative for small τ . For large φ consider $v(1) = u(1, 1 - \tau, 1) - u(\tau, 1, 1) - u(1 - \tau, 0, 1) = u(1, 0, 1) - u(\tau, 0, 1) - [u(1 - \tau, 0, 1) - u(0, 0, 1)]$ so that $v(1) > 0$ if $u(y_a, 0, 1)$ is strictly convex in y_a . To see why this must be the case, notice that when $\varphi = 1$ defensive effort does not matter, the defender optimally chooses $x_d = 0$ and the aggressor faces the simple problem of maximizing $(\Pi_\zeta(\eta) + \alpha\pi_0(x_a, 0, 1))y_a - c(y_a)$. The first and second order conditions for the optimum together with the implicit function theorem show that the solution of this problem is a convex function of y_a and therefore $u(y_a, 0, 1)$ is convex as well.

A second fact about the model is that conflict is rare and the expected time between conflicts long. This has two consequences. First, the random choice of active group does not much matter. It might be that under hegemony the prospective rebels draw two independent utility shocks one for inclusive and one for extractive institutions - if the larger of the two resulting utilities is positive then a rebellion attempting to install the “more profitable” institutions is launched. However: since the probability either is positive is very small the chance that both are simultaneously positive is negligible and we might as well assume one is chosen at random as a prospective agenda. Similarly in a balance of power each society may simultaneously draw independent utility shocks: again the chance both are positive simultaneously is negligible. A more technical way to say this is that the resistance to a single positive shock is much less than the resistance to a simultaneous shock, so least resistance transitions involve only a single positive shock.

The other consequence of conflict being rare is that the model is robust to the assumption that groups are myopic. If we interpret the utility shocks as the long-term present value of the effect of conflict (death, disruption of social organizations, destruction of capital, political popularity and so forth) then groups may as well discount the future with a discount factor $\delta < 1$. With high probability another “good” draw of the utility shock will be a long time in coming so that in effect the decision is a static one: stay with the status quo for a long time, or attempt to switch to an alternative which if successful will also last a long time.

In a similar vein: we might wonder whether occupier elite might not choose to set taxes lower

than the most that they can extract in hopes of reducing the chance of rebellion. Since the chances of rebellion are very small and the tax reduction would have to be permanent this would not make sense.

Finally, continuity considerations imply that the model is robust with respect to perturbations that have a small effect on resistances.¹¹ For example, while it makes sense that in an inclusive society occupier masses take the decisions and in an extractive state occupier elites do, for the occupied society the situation is less clear cut. Let y_m and y_e denote the utility gain in a hegemony to the occupied masses and occupied elite from a rebellion (with respect to some already determined goal). The basic model assumes that if the hegemony is inclusive only y_m the incentives of the masses matter, while if the hegemony is extractive only y_e the incentives of the elites matter. Consider instead a model where in an inclusive hegemony a weighted average $(1 - \omega)y_m + \omega y_e$ of the gain to each group represents the “overall gain” to rebellion, and likewise in an extractive hegemony $\omega y_m + (1 - \omega)y_e$ represents the “overall gain.” Simple continuity considerations show that the qualitative results of theorem 1 are robust to ω slightly positive. In fact ω can be quite large: instead of assuming $\omega = 0$ assume only that $\omega < 1/2$ to reflect that in an inclusive society masses do have greater political strength than elites and vice versa. In the Web Appendix we show that the qualitative conclusions of the model are robust to this perturbation. The structure of circuits remains unchanged and it remains the case that only s , ss and ww can be stochastically stable. The cutoff for an extractive hegemony over an extractive balance of power $\eta^*(\varphi, \omega)$ now depends on the military technology and on ω , and the specific numeric value of the cutoff τ^* changes: but we show that when $\tau = 0$ and $\varphi > 0$ then ww has a greater modified radius than ss and that when $\varphi = 0$ and $\tau > 0$ then ss has greater modified radius than ww .

This is an appropriate place to comment on a key driving force in the model: that $r(w \rightarrow sw) = U - \Pi_h(\eta) - u(0, 0, \varphi)$ and $u(0, 0, \varphi) = 0$ implying very high resistance to moving from an inclusive hegemony to a balance of power where rebellion has led to extractive institutions. In the basic model this is due to the fact that political power among the rebels lies with the masses who have no reason to replace extractive foreign institutions with extractive domestic institutions ($y_a = 0$). It might seem that the masses being indifferent and the elite standing to gain 1 by rebellion the masses might go along with the elites so that in fact $y_a = 1$. Our analysis of $\omega < 1/2$ shows that this is not true. Why not? The point is that $y_a = 0$ does *not* imply the masses are indifferent. Their utility gain is 0 *plus* the utility shock which is with high probability very negative. If we assume that a common utility shock is drawn by the masses and the elite then because the probability of larger shocks drops very rapidly the most likely scenario in which the elite wish to rebel is one in which they have only a slight preference for doing so. On the other hand a utility shock that leaves the elite slightly preferring to rebel will have the masses rather more strong preferring not to rebel: it is not the cases that the masses are indifferent so should defer to the wishes of the elite, rather

¹¹Perturbations such as a random determination of whether masses or elites are in control that have a discontinuous effect on resistances and lead in general to entirely different conclusions.

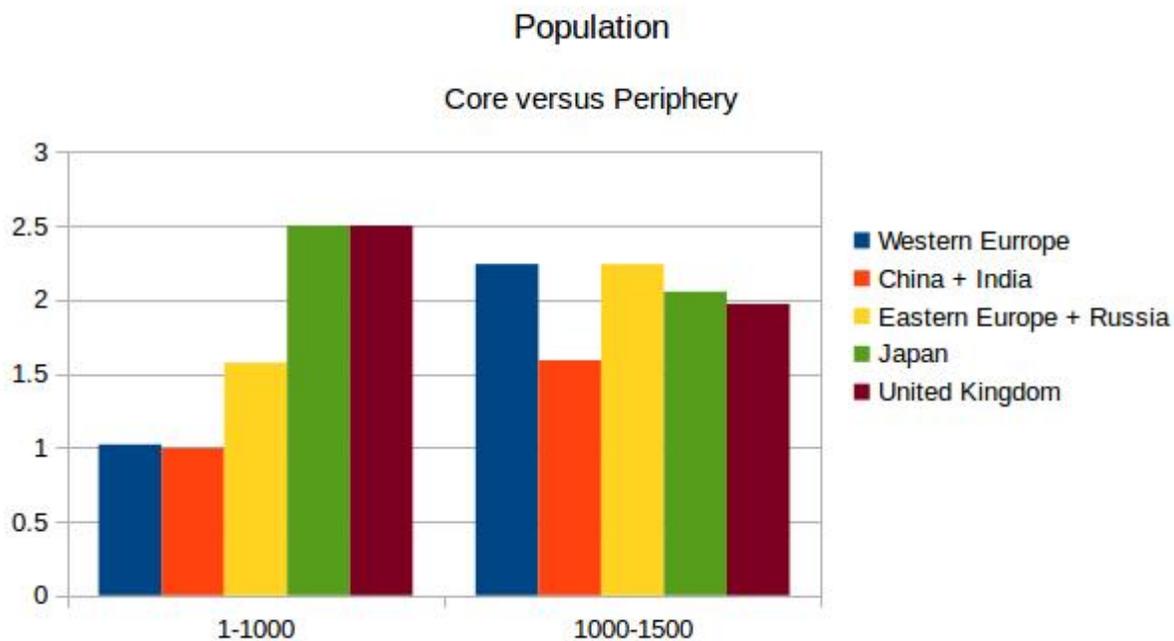
the opposite. Our analysis in the web appendix of the full model shows, however, that this is not enough: some slight political advantage must lie with those favored by the dominant institutions to generate the qualitative features of the basic model.

4. History

The theory suggests that institutions and international competition can be explained by two variables: the strength of outsiders and military technology. In particular, weak outsiders should result in extractive hegemonies regardless of military technology. Strong outsiders should result in a balance of power: between extractive states if fixed defenses are strong and between inclusive states if fixed defenses are easily overcome. We are interested also in the implications for innovation. We accept the basic premise that inclusive institutions lead to greater innovation than extractive institutions: for recent evidence on this point see Serafinelli and Tabellini (2017). Here we see whether these basic ideas are useful in organizing the history of Europe and Asia during the Common Era. We focus on the main centers of population in this period: Europe, China and India.

Population

The demographic record of the Common Era is marked by what we would call early globalization: the rising population of peripheral areas relative to the core areas of civilization. Using population data from the Maddison Project Maddison (2013) the table below records the growth of population in different regions in the period 1-1000 CE and in the period 1000-1500 CE. It can be seen that in the earlier period the population in the core areas (Western Europe, China + India) grew at a similar rate while peripheral areas including Eastern Europe, Britain and Japan grew much more quickly. In the later period growth rates are similar in the core and periphery.



This limited data is consistent with what we know more broadly: the massive movement of Germanic peoples from Central Asia to Northern Europe around 200 CE is well known - and their influence in the decline and fall of the Roman Empire well documented. In China a similar event occurs with the arrival of the five barbarians around 300-400 CE while in the 400s the Alchon Huns arrived in India from Central Asia. It is probably accurate to say that population centers that had been largely separate (Central Asia, Europe, China, India) began to collide as population grew in all of these areas.

Against this general relative increase in peripheral population must be set one important demographic event: the Mongolian diaspora around 1200 CE. While the demographics of Mongolia either before or after Genghis Khan are not well documented there seems little doubt that the conquests of Genghis Khan and his generals extending as they did across Europe and Asia resulted in a substantial movement of people away from Mongolia and towards China, Eastern Europe and Central Asia - reversing to an extent the earlier migration out of Central Asia.

A Brief History of Siege Warfare

Every tourist has noticed that a feature of ancient cities absent from modern cities are the existence of walls. There is archaeological evidence of brick city walls as early as 2500 BCE (Fletcher and Cruikshank (1996)). Since that time there has been a race between the technology of walls and the technology of siege: better design and construction techniques for walls have competed with the development of battering rams, ladders, siege towers, ballistas and catapults. With the development of the cannon the technology of walls gradually became obsolete: city walls in the current time would be completely useless for defensive purposes as modern artillery would reduce them to ruin in a matter of moments. In World War II the immense and sophisticated Belgium fortress of Eben-Emael was rendered ineffective by a German force of 75 men in a few hours (Kauffmann and Jurga (2002)).

The advantage of effective fortifications is that they enable a small army to hold off a much larger force. In Masada in 66 CE, for example, a group of roughly 1,000 men women and children held off the Roman Empire for about seven years before being overcome by a military force of around 15,000. On the other hand, effective fortifications reduce the benefit of a larger defending force: it is unlikely that a Jewish force of 2,000 or 3,000 would have had much more success against the Romans than 1,000. In terms of our model we measure the effectiveness of defense by $(1 - \varphi)x_d + \varphi$ where x_d is the strength of the defenders and φ measure the strength of the fortification: this capture the basic idea that with effective fortifications the defense is strong but not particularly sensitive to defensive strength.

The theory directs us to investigate the effectiveness of fortifications over time. Until the advent of the cannon it does not appear to have changed greatly at which point it began to deteriorate gradually as improvement in cannons greatly outstripped improvements in fortifications. The story of the cannon begins in China with the invention of gunpowder sometime before 808 CE (Unknown (808)). Gunpowder technology did not reach Europe until some 400 years later probably from the Mongols: the first hard evidence we have is the description in Bacon (1276) of the manufacture and

use of gunpowder. Several hundred years after that cannons were in widespread use as effective siege weapons: by the 1400s they effectively ended the era of castles smashing the walls of Byzantium and the Moorish fortifications in southern Spain.

Starting earlier China saw a more gradual improvement in siege technology. Long before the invention of the cannon (there are reports of a cannon be used in siege in 1132) gunpowder was incorporated in a variety of weapons: a military manual (Gonglang, Du and Weide (1044)) from early in the Song dynasty (1044 CE) describes a variety of these. What proved most important in practice was the development of the bomb. Siege technology prior to gunpowder involved the use of catapults and ballistas of various types to hurl stones at or over city walls. Replacing stones with bombs proved extremely effective. In the 1100-1200s during the various wars between the Jin and Song there was an R&D race with each side vying to develop improved explosives and holding a temporary military edge over the other as they developed larger and more explosive bombs. Large bomb factories were built and bombs produced in large quantities by both sides and we read, for example, descriptions of armies as having three thousand men and three thousand thunderclap bombs (Andrade (2016)).

The story in China and Europe then is gradually declining fortification strength φ beginning in China around 1000 CE and in Europe around 1300 CE. In India the story is different. Gunpowder and cannons seem to have reached India only with the Moghuls: according to Baburnama, Habib and Baipakov (2004) sometime after 1496. Roughly speaking by the time the Indians adopted cannons it was too late: the foreign invaders had arrived and the second era of globalization - colonization - arrived. The decisive Portuguese sea victory at the Battle of Diu in 1509 CE established long term European military dominance over the sea and Babar's conquest of India is usually agreed to have been effectively ended in 1526 CE.

Europe

Europe saw increased outsiders followed by improved siege technology. In the earliest period 1 CE - 330 CE the Roman Empire formed an extractive hegemony - and one that provided little in the way of innovation.¹² In 330 CE the Empire was split in half and until about 1293 CE we have the early medieval period which may well be described as an extractive balance of power in which warlords rivaled one another to squeeze the peasant. This era is famous for its lack of innovation, having earned the sobriquet: the "dark ages." For our purposes we might take the publication of the Ordinances of Justice in Florence in 1293 CE as an indication of inclusive institutions and use that date as the beginning of subsequent era: from this period on there is typically a balance of power but among more inclusive societies. There is substantial innovation during this late medieval period culminating in the industrial revolution.

The theory explains events in Europe relatively well. Initially with few outsiders there is an extractive hegemony. As outsiders arrive this shifts to an extractive balance of power. Finally as

¹²Roman innovations in water management, transportation and construction were considerable but appear to originate in the earlier period of the Republic.

military technology develops and siege technology improves we see primarily an inclusive balance of power.

China

The story of China is more complicated than Europe. The early Han dynasty began in 206 BCE and lasted through the beginning of the Common Era until 25 CE: it can be well described as an extractive hegemony. It was then supplanted by the later Han dynasty from 25 CE - 220 CE. This era is famous in Chinese history both for its innovation as well as liberal institutions (for the time). It can reasonably be described as an inclusive hegemony. From 220 CE - 581 CE there was a long period of warlordism that may well be described as an extractive balance of power. This is followed in 581 CE - 907 CE by the extractive hegemonic Sui and Tang dynasties. In 907 CE - 960 CE there are the Five Dynasties and Ten Kingdoms followed by Song dynasty in 960 - 1279 CE: the latter we would generally describe as an inclusive balance of power. We emphasize in particular that the Song dynasty held a hegemony at best very briefly: it was in a balance of power first with the Liao dynasty subsequently the Xia and Jin. The Song era had relatively progressive institutions - in particular important reforms were made to an expanded imperial examination system. It was renowned for its innovation: some of the highlights are the invention of canal locks, paddle boats, maritime technology various measurement devices, improvements in the use of power - and in addition to crossbows, and most notably the invention of gunpowder together with land mines, bombs, flamethrowers and of course cannons. After 1279 CE China reverts to extractive hegemony with the Yuan, Ming and Qing dynasties - none particularly famous for innovation.

To understand how our theory can and cannot account for this history it is important to remember that the theory predicts what will be seen “most of the time.” Hence while the theory does not predict we should see a later Han dynasty with inclusive hegemony, it says we might see it but it should be relatively short lived. And indeed it lasted 200 years - as compared to the nearly 400 years of the Song era and the 700 years of extractive hegemony and extractive balance of power that preceded the Song era. So while not confirming the theory the later Han dynasty does not contradict it.

Leaving aside the Han, the period prior to the Song era shifts back and forth between extractive balance of power and extractive hegemony, each about 350 years in duration. Lacking adequate demographic data we can conjecture that during this period the strength of outsiders had risen to roughly η^* meaning that we should observe both types of institutions. Then, with the improvement of siege technology, as the theory predicts we move to the inclusive balance of power of the Song era. Finally the adverse demographic event of the Mongolian diaspora lowered the strength of outsiders sufficiently that reversion to extractive hegemony occurred.

India

The story of India can be described as too soon and too late: the outsiders arrived too soon for extractive hegemony and cannons arrived at the same time as the Europeans - too late for inclusive balance of power. Indeed: until conquest by the British India had an extractive balance

of power through most of its history. India also has had remarkably little innovation: it is famous for advances in art, architecture, mathematics and astronomy - but not in the more practical arts.

The late arrival of the cannon in India we indicated above. The key aspect of Indian history we point to is evidence that there were always strong outsiders. These were the Central Asians. First, from the perspective of population in 1820: the population of Mongolia was only about 20% of the population of Afghanistan, while in 1950 the Soviet “stans” (Kazakhstan Kyrgyzstan Tajikistan Turkmenistan) were in aggregate about 40% larger than Afghanistan - suggesting that the relevant population of outsiders was much larger for India than for China (both countries being of similar population throughout the CE). That Central Asians were outsiders - and strong outsiders - is indicated by the repeated successful invasion of India from Central Asia - and the absence of any successful invasion of Central Asia from India. As early as 520 BCE the Persians occupied parts of India. Around around 100 CE the Parthians invaded successfully followed by the Kushans in about 320 CE. Unlike Rome and China which faced few and weak outsiders in their early history India always did so.

Early History

Although our focus is on the Common Era - for the obvious reason of data availability - the BCE history of Egypt, China and Persia is of interest. In this era prior to early globalization great hegemonies spanned centuries.

In Egypt during the 1,617 years from 2686 BCE to the end of the new Kingdom in 1069 BCE there are roughly 500 years of “intermediate period” during which there is not an extractive hegemony (Shaw (2000)). It is also the case that the decline of Egypt coincides with the arrival of the historically mysterious sea people in roughly 1200-900 BCE. They may or may not have been successful invaders: our theory indicates they need not have been - the strengthening of outsiders is in general inimical to hegemony.

In China during the 1820 years from 1600 BCE to 220 CE there are 501 years of Spring and Autumn and warring states period - the remaining being extractive hegemony Wright (2006). In Persia during the 1,201 years from 550 BCE to 651 we have extractive hegemony except for about 87 years between the Achaemenid Empire and Parthian/Sassanid Empires Daryaee (2012).

The basic point is that Before Common Era when outsiders were less strong hegemonies were indeed more frequent.

Concluding Observations

It would be ludicrous to suggest that a model with five states and eight possible transitions could fit data over a period of 2000 years in three very different regions of the world on some sort of minute-by-minute basis. There are many things that the model does not account for: for example, in reality power does not shift abruptly between elite or masses - rather there is internal evolution of political systems. There are of course many more than two societies and two groups in each society. Rather we have argued a simple model that captures in a broad way the impact

on incentives of the external environment (strength of outsiders) and technology (gunpowder in particular) can provide a reasonable historical narrative.

On the other hand the model helps to direct our thinking. For example: it is widely recognized that the period of the renaissance is crucial in leading up to the industrial revolution. Many have suggested that the black death plague was instrumental in shifting economic strength from elites to masses in Europe. This may well be true. But the black death plague was spread by the Mongolians to China as much as to Europe yet had no beneficial effect there. The model directs us to the alternative possibility that the black death was not the important consequence of Genghis Khan, but rather it was the fact he brought gunpowder to Europe and depopulated Mongolia.

The model also leads us to distinguish between outsiders who invade, displace inside elites, and by doing so reduce outside pressure, and outsiders who do not do so. Consider that we have argued that hegemony in China after 1200 is largely due to the depopulation of Mongolia. That hegemony remained, but was overthrown by outsiders twice. For example, the Qing dynasty replaced the Ming in 1644 when the Manchurians displaced the Ming elite. On the one hand it can be said that Manchuria conquered China. On the other hand it is equally true that China conquered Manchuria in the sense that the Qing dynasty ruled over Manchuria - so that Manchurians were no longer outsiders and so there was little pressure against hegemony. Why did not the same thing happen when the Germanic tribes overcame the Roman Empire? That is, just as there were Mongol and Manchurian emperors of China, so there were Germanic Roman emperors. The model provides a straightforward answer: the Germanic invasions of the Roman Empire did not depopulate Eastern Europe nor did Eastern Europe become part of the Roman Empire - strong outsiders remained. The point is that the Germanic invasions and settlement of Roman territories were driven by population pressure from the East: the Germans outsiders were themselves being displaced by other outsiders. The same was not true in Mongolia and Manchuria.

In a sense our theory relies also on geographical factors. Unlike Diamond (1998) who suggests that internal features of terrain are important, our theory suggests that the relatively hospitable climate of Eastern Europe and Central Asia played an important role in leading to different historical outcomes in Western Europe and India compared to the role of the relatively inhospitable climate of northern Asia in Chinese history.

5. Conclusion

In this paper we develop a simple stylized model of conflict between societies. There are two societies contending over resources and one may or may not rule over the other. There are two groups: masses and elites. Institutions can be either inclusive or extractive with inclusive institutions substantially more favorable for the masses. We examine stochastic stability and show that when outsiders are weak extractive hegemonies will predominate. When outsiders are strong military technology matters: if outcomes are insensitive to defensive strength - fortifications are well able to resist siege - an extractive balance of power will predominate, while good siege technology (the cannon in particular) will result in the predominance of an inclusive balance of power.

We apply this theory historically. We argue that generally speaking the importance of outsiders has increased over time and siege technology has improved. We document that the increased importance of outsiders in Europe and India preceded the advent of good siege technology: our theory implies that we should see an extractive hegemony followed by an extractive balance of power followed by an inclusive balance of power. In Europe this corresponds roughly to the Roman Empire, followed by the early medieval period followed by the renaissance. In India we argue that outsiders were always strong and that good siege technology arrived “too late” so that in India prior to conquest by Britain there was extractive balance of power. In China we see a back on forth between extractive hegemony and extractive balance of power until the relatively early arrival of good siege technology coincides with the flowering of the Song and rival states. However a striking demographic event - the Mongolian diaspora - greatly weakened the strength of outsiders resulting in a reversion to extractive hegemony.

We examine also the implications for innovation. Here the distinction between an extractive and inclusive balance of power plays a key role. We hypothesize that only the latter is favorable for innovation - hence while we see a great deal of competition between societies both in early medieval Europe and in India we do not see a great deal of innovation. We attribute this to the fact that this was competition between extractive societies - while the competition in Europe beginning with the renaissance and between the Song dynasty and its rivals in China was competition between inclusive societies that consequently generated substantial innovation.

The notion that the growth of modern inclusive societies was driven by the “democratization” of warfare is not a new one; nor, of course is the idea that outsiders destroy hegemonies. Our account is a more subtle one: we see that “democratization” of warfare as being driven by the decline of the fortress because of the cannon rather than the introduction of hand held fire-arms. We also see the impact of outsiders not so much in conquering empires but in weakening their resistance to revolution and strengthening the balance of power. This more subtle theory appears to do a reasonably job of organizing the historical data.

References

- Acemoglu, Daron and James A. Robinson (2012): *Why Nations Fail: The Origins of Power, Prosperity, and Poverty*, Crown Publishing Group.
- Andrade, Tonio (2016): *The Gunpowder Age: China, Military Innovation, and the Rise of the West in World History*, Princeton University Press.
- Baburnama, AC, I Habib and KM Baipakov, eds. (2004): *History of Civilizations of Central Asia: Development in Contrast : from the Sixteenth to the Mid-Nineteenth Century*, United Nations Educational.
- Bacon, Roger (1276): *Opus Majus*.
- Besley, Timothy and Torsten Persson (2010): "State Capacity, Conflict and Development," *Econometrica* **78**: 1-34.
- Bisin, A., and T. Verdier (2001): "The economics of cultural transmission and the dynamics of preferences," *Journal of Economic Theory*, 97: 298-319.
- Bisin, A., and T. Verdier (2005): "Cultural Transmission," *The New Palgrave Dictionary of Economics*.
- Bowles, Samuel and Jung-Kyoo Choi (2013): "Coevolution of farming and private property during the early Holocene" *Proceedings of the National Academy of Science*, doi:10.1073/pnas.1212149110
- Choi, Jung-Kyoo and Samuel Bowles (2007): "The coevolution of parochial altruism and war," *Science* 318: 636-40.
- Cipolla, Carlo (1965): *Guns, Sails, and Empires: Technological Innovation and the Early Phases of European Expansion, 1400- 1700*.
- Daryaee, Touraj (ed)(2012): *Persia Oxford Handbook of Iranian History*, Oxford University Press.
- Diamond, Jared (1998): *Guns, Germs, and Steel: The Fates of Human Societies*.
- Dutta, R., D. K. Levine, N. W. Papageorge, and L. Wu (2018): "Entertaining Malthus: Bread, Circuses, and Economic Growth," *Economic Inquiry*, 56(1), 358-380.
- Ellison, Glenn (2000): "Basins of Attraction, Long Run Stochastic Stability and the Speed of Step-by-step Evolution," *Review of Economic Studies* 67: 17-45.
- Fletcher, Banister and Cruickshank, Dan (1996): *Sir Banister Fletcher's A History of Architecture* (20th ed.). Architectural Press.
- Garfinkel, Michelle and Stergios Skaperdas (2007): "Economics of Conflict: An Overview," *Handbook of Defense Economics*, Elsevier.
- Gongliang, Zeng, Ding Du and Yang Weide (1044): *Wujing Zongyao*.
- Greif, A., and G. Tabellini, G. (2010): "Cultural and institutional bifurcation: China and Europe compared," *American Economic Review*, 100: 135-40.
- Hausken, Kjell (2005): "Production and Conflict Models Versus Rent Seeking Models," *Public Choice* 123: 59-93.
- Hayek, F. A. (1944): *The Road to Serfdom*, Dymock's Book Arcade.
- Herrera, H., M. Morelli, and S. Nunnari (2015): "Turnout Across Democracies," *American Journal of Political Science*.
- Hirshleifer, Jack (2001): *The dark side of the force: economic foundations of conflict theory*, Cambridge University Press.
- Hoffman, Philip T. (2013): *Why Did Europe Conquer the World?* forthcoming Princeton University Press.
- Kauffmann, J.E. and R. Jurga (2002): *Fortress Europe: European Fortifications of World War II*, Da Capo Press, USA.

- Kandori, Michihiro, George Mailath, and Rafael Rob (1993): "Learning, Mutation, and Long Run Equilibria in Games," *Econometrica* 61: 29-56.
- Landes, D. S. (2003): *The unbound Prometheus: technological change and industrial development in Western Europe from 1750 to the present*, Cambridge University Press.
- Levine, David and Salvatore Modica (2012): "Conflict and the Evolution of Societies," working paper, www.dklevine.com.
- Levine, David and Salvatore Modica (2013): "Anti-Malthus: Conflict and the Evolution of Societies," *Research in Economics* 67: 289-306
- Levine, David and Salvatore Modica (2016): "Dynamics in stochastic evolutionary models", *Theoretical Economics* 11: 89-131
- Levine, David and Salvatore Modice (2017) "An Evolutionary Model of Intervention and Peace", working paper www.dklevine.com
- Lin, Justin (January 1995). "The Needham Puzzle: Why the Industrial Revolution Did Not Originate in China". *Economic Development and Cultural Change*. 43 (2): 269–292
- Liu, Yingqui; Chunjiang Liu (2007). "Diagnosing The Cause of Scientific Standstill, Unravelling The Needham Puzzle". *China Economist*. 10: 83–96.
- Maddison, Angus (1998): *Chinese Economic Performance in the Long-Run*, OECD Development Centre, Paris.
- Maddison, Angus (2013): *Historical Statistics for the World Economy: 1-2003 AD*, spreadsheet available at www.gdpc.net.
- McNeil, William (1963): *The Rise of the West*, University of Chicago Press.
- Mokyr, J. (2002): *The gifts of Athena: Historical origins of the knowledge economy*, Princeton University Press.
- Morris, I. (2010): *Why the west rules - for now: The patterns of history and what they reveal about the future*, Profile books.
- Olson, M. (1982): *The rise and decline of nations: Economic growth, stagflation, and social rigidities*, Yale University Press.
- Rogers D. S., O. Deshpande and M. W. Feldman (2011): "The Spread of Inequality," PLoS ONE 6.
- Rosenthal, J. L., and R. B. Wong, R. B. (2011), *Before and Beyond Divergence*, Harvard University Press.
- Saturnino, Monteiro (2011): *Portuguese Sea Battles Volume I – The First World Sea Power*
- Serafinelli, M., and Tabellini, G. (2017), "Creativity over time and space," working paper, Bocconi.
- Shaw, Ian (ed) (2000): *The Oxford History of Ancient Egypt*: Oxford University Press.
- Sheehan, Michael J. (1996): *The Balance of Power: History and Theory*, Taylor and Francis.
- unknown (808): *Taishang Shengzu Jindan Miju*.
- Wright, Edmund (2006): *A Dictionary of World History*, second edition, Oxford University Press.
- Young, Peyton (1993): "The Evolution of Conventions," *Econometrica* 61: 57-84.

Appendix: The Conflict Subgame

Theorem. [Theorem 2 in the text] The conflict subgame has a unique Nash equilibrium independent of the state and strength of outsiders. The total utility gain to the aggressor $\Pi_\zeta(\eta)y_a + u(y_a, y_d, \varphi)$ is non-negative, less than or equal to one and strictly increasing in y_a . The partial utility gain $u(y_a, y_d, \varphi)$ is strictly increasing in φ , decreasing in y_d , and satisfies $u(0, y_d, \varphi) = 0$. There is a $0 < \tau^* < 1$ and a strictly decreasing function $0 < \varphi_\tau < 1$ such that the function $v(\varphi) \equiv u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi) - u(1 - \tau, 0, \varphi)$ satisfies $|v(\varphi)| < \alpha$ with $v(\varphi) < 0$ for $\tau < \tau^*$ and $\varphi < \varphi_\tau$ and $v(\varphi) > 0$ otherwise.

Proof. Recall that $0 \leq x_a, x_d \leq 1$, $0 \leq y_a, y_d \leq 1$. The objective functions for the aggressor and for the defender are respectively

$$\begin{aligned} & \Pi_\zeta(\eta)y_a + \alpha(-\varphi + (x_a - (1 - \varphi)x_d))y_a - (\gamma/2)x_a^2 \\ \text{and } & [1 - \Pi_\zeta(\eta) - \alpha(-\varphi + (x_a - (1 - \varphi)x_d))]y_d - (\gamma/2)x_d^2. \end{aligned}$$

In this simple linear quadratic model the optimal choice for aggressor and defender are independent of each other, so the equilibrium is certainly unique. Set $\beta = \alpha/\gamma$. The optimum for the aggressor is $\hat{x}_a = \beta y_a$ and for the defender is $\hat{x}_d = \beta(1 - \varphi)y_d$. Observe that we certainly have $\alpha < 1/2$ while $\gamma \geq 1$ so that $\beta \leq 1$ and these are interior solutions. Hence the partial utility gain to the aggressor is given by

$$\begin{aligned} u(y_a, y_d, \varphi) &= \alpha(-\varphi + (\beta y_a - \beta(1 - \varphi)^2 y_d))y_a - (\gamma/2)\beta^2 y_a^2 \\ &= \alpha[(-\varphi - \beta(1 - \varphi)^2 y_d)y_a + (1/2)\beta y_a^2]. \end{aligned}$$

Observe that the total utility gain is $[\Pi_\zeta(\eta) - \alpha(\varphi + \beta(1 - \varphi)^2 y_d)]y_a + (\alpha/2)\beta y_a^2$ and that $\Pi_\zeta(\eta) - \alpha(\varphi + \beta(1 - \varphi)^2 y_d) \geq \Pi_\zeta(\eta) - \alpha > 0$ by assumption. Hence the total utility gain is non-negative and strictly increasing in y_a .

We next observe that $u(y_a, y_d, \varphi)$ is decreasing in y_d and satisfies $u(0, y_d, \varphi) = 0$. We compute $v(\varphi) \equiv u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi) - u(1 - \tau, 0, \varphi)$. The computation gives

$$v(\varphi) = \alpha\beta((1 - \varphi)^2(2\tau - 1) + (1 - \tau)\tau).$$

Hence $-\alpha \leq v(\varphi) \leq \alpha(-\tau^2 + 3\tau - 1) \leq \alpha$ for $0 \leq \tau \leq 1$. If $2\tau - 1 > 0$ clearly $v(\varphi) > 0$ for all φ . For $2\tau - 1 < 0$ the function $v(\varphi)$ is increasing in φ and we may solve $v(\varphi_\tau) = 0$ to find

$$\varphi_\tau \equiv 1 - \sqrt{\frac{(1 - \tau)\tau}{1 - 2\tau}}.$$

This is obviously strictly less than 1. It is positive if $q(\tau) = 1 - 2\tau - (1 - \tau)\tau > 0$. From the quadratic equation we find $q(\tau^*) = 0$ for $\tau^* \approx 0.38 < 1/2$. For $\tau > \tau^*$ we have $q(\tau) < 0$ hence $v(\varphi) > 0$ for all φ . When $\tau < \tau^*$ then $0 < \varphi_\tau < 1$ and for $\varphi > \varphi_\tau$ we have $v(\varphi) > 0$ while for

$\varphi < \varphi_\tau$ we have $v(\varphi) < 0$. Finally, we compute $\frac{d\varphi_\tau}{d\tau} \propto -\frac{(1-2\tau)^2+2(1-\tau)\tau}{2(1-2\tau)} < 0$. □

Web Appendix

We study the situation where in an inclusive hegemony a weighted average $(1 - \omega)y_m + \omega y_e$ of the gain to each group represents the “overall gain” to rebellion, and likewise in an extractive hegemony $\omega y_m + (1 - \omega)y_e$ represents the “overall gain.” Instead of assuming $\omega = 0$ as in the basic model assume only that $\omega < 1/2$.

Theorem. *The structure of circuits is the same as in the basic model and it remains the case that only s, ss and ww can be stochastically stable. The cutoff for an extractive hegemony over an extractive balance of power is $\eta^*(\varphi, \omega)$ which is larger than the η^* of the basic model. When $\tau = 0$ and $\varphi < 1$ then ww has a greater modified radius than ss ; when $\varphi = 1$ then ss has greater modified radius than ww .*

Proof. We modify the resistance table used in the proof of theorem 1 to account for utility of rebels that is a weighted average of that of the masses and that of the elites. As before we indicate first the transition then the benefit from the transition to the masses and elites respectively. This is divided into two columns for occupier or occupied where as in the text, if the initial state is a hegemony these are the hegemony’s four groups; in case the initial state is a balance of power occupier versus occupied refers to the situation after transition. The difference is that now for example in the transition $s \rightarrow sw$, we had $y_a = \tau$ - the elite’s gain - we now have the average $(1 - \omega)y_e + \omega y_m = (1 - \omega)\tau + \omega(1 - \tau)$. As before the asterisks denote which group is making the decision about attack and paying for defense and the final column reports the resistance.

transition	occupier	occupied	$r(z \rightarrow z')$
$ss \rightarrow s$	$0, 1^*$	$0, -1^*$	$U - \Pi_b(\eta) - u(1, 1, \varphi)$
$s \rightarrow ss$	$0, -1^*$	$0, 1^*$	$U - \Pi_h(\eta)(1 - \omega) - u(1 - \omega, 1, \varphi)$
$s \rightarrow sw$	$0, -1^*$	$1 - \tau, \tau^*$	$U - \Pi_h(\eta) ((1 - \omega)\tau + \omega(1 - \tau)) - u((1 - \omega)\tau + \omega(1 - \tau), 1, \varphi)$
$sw \rightarrow s$	$0, 1^*$	$-(1 - \tau)^*, -\tau$	$U - \Pi_b(\eta) - u(1, 1 - \tau, \varphi)$
$sw \rightarrow w$	$0^*, 1$	$0, -1^*$	$U - u(0, 1, \varphi)$
$w \rightarrow sw$	$0^*, -1$	$0^*, 1$	$U - \Pi_h(\eta)\omega - u(\omega, 0, \varphi)$
$w \rightarrow ww$	$0^*, -1$	$1 - \tau^*, \tau$	$U - \Pi_h(\eta) ((1 - \omega)(1 - \tau) + \omega\tau) - u((1 - \omega)(1 - \tau) + \omega\tau, 0, \varphi)$
$ww \rightarrow w$	$0^*, 1$	$-(1 - \tau)^*, -\tau$	$U - u(0, 1 - \tau, \varphi)$

We again start by determining the structure of circuits. We observe that

$$\begin{aligned} r(s \rightarrow sw) &= U - \Pi_h(\eta) ((1 - \omega)\tau + \omega(1 - \tau)) - u((1 - \omega)\tau + \omega(1 - \tau), 1, \varphi) \\ &> U - \Pi_h(\eta)(1 - \omega) - u(1 - \omega, 1, \varphi) = r(s \rightarrow ss) \end{aligned}$$

for $\omega < 1 - \omega$ by the monotonicity property of $u(y_a, y_d, \varphi)$. This implies that ss and s form a circuit. Also by monotonicity we have $r(w \rightarrow ww) = U - \Pi_h(\eta) ((1 - \omega)(1 - \tau) + \omega\tau) - u((1 - \omega)(1 - \tau) + \omega\tau, 0, \varphi) < U - \Pi_h(\eta)\omega - u(\omega, 0, \varphi) = r(w \rightarrow sw)$ for $\omega < 1 - \omega$ so that ww and w form a circuit. Since the resistance at a balance of power is unchanged the resistance of joining the

$ss \leftrightarrow s$ circuit is still less than joining the $w \leftrightarrow ww$ circuit. Hence circuits are still described by the diagram $[(ss \leftrightarrow s) \leftrightarrow sw] \leftrightarrow (w \leftrightarrow ww)$.

We next revise the table of modified resistances. The second column shows how the modified radius depends upon the resistances, the third how they depend on the model parameters. In the computation we make use of the fact that $u(0, y_d, \varphi) = 0$.

state	R	R
ss	$r(ss \rightarrow s) - r(s \rightarrow ss) + r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$\Pi_h(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi) + R(s)$
s	$r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U - \Pi_h(\eta)((1 - \omega)\tau + \omega(1 - \tau)) - u((1 - \omega)\tau + \omega(1 - \tau), 1, \varphi) + \Pi_b(\eta) + u(1, 1 - \tau, \varphi)$
sw	$r(sw \rightarrow w)$	U
w	$r(w \rightarrow sw)$	$U - \Pi_h(\eta)\omega - u(\omega, 0, \varphi)$
ww	$r(ww \rightarrow w) - r(w \rightarrow ww) + r(w \rightarrow sw)$	$U + \Pi_h(\eta)((1 - \omega)(1 - \tau) + \omega\tau) + u((1 - \omega)(1 - \tau) + \omega\tau, 0, \varphi) - \Pi_h(\eta)\omega - u(\omega, 0, \varphi)$

As before $R(ww) > R(w)$, $R(sw)$ so these two states are not stochastically stable. We compare

$$R(ss) - R(s) = \Pi_h(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi).$$

We seen then that $\eta^*(\varphi, \omega)$ is the unique solution from equating this to zero. Notice that

$$\Pi_b(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi) < 0$$

so that $\Pi_h(\eta^*(\varphi, \omega)) - \Pi_b(\eta^*(\varphi, \omega)) > 0$ implying that fewer outsiders are needed to tip towards a balance of power than in the basic model.

Next consider $\tau = 0$. We have

state	R	R
ss	$r(ss \rightarrow s) - r(s \rightarrow ss) + r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$\Pi_h(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi) + R(s)$
s	$r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U - \Pi_h(\eta)\omega - u(\omega, 1, \varphi) + \Pi_b(\eta) + u(1, 1, \varphi)$
ww	$r(ww \rightarrow w) - r(w \rightarrow ww) + r(w \rightarrow sw)$	$U + \Pi_h(\eta)(1 - \omega) + u((1 - \omega), 0, \varphi) - \Pi_h(\eta)\omega - u(\omega, 0, \varphi)$

from which

$$\begin{aligned} R(ss) - R(ww) &= \Pi_h(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi) \\ &\quad - \Pi_h(\eta)\omega - u(\omega, 1, \varphi) + \Pi_b(\eta) + u(1, 1, \varphi) \\ &\quad - \Pi_h(\eta)(1 - \omega) - u(1 - \omega, 0, \varphi) + \Pi_h(\eta)\omega + u(\omega, 0, \varphi) \\ &= u(1 - \omega, 1, \varphi) - u(\omega, 1, \varphi) - u(1 - \omega, 0, \varphi) + u(\omega, 0, \varphi). \end{aligned}$$

From $u(y_a, y_d, \varphi) = \alpha [(-\varphi - \beta(1 - \varphi)^2 y_d) y_a + (1/2)\beta y_a^2]$ we see that

$$R(ss) - R(ww) = -\alpha\beta(1 - \varphi)^2(1 - 2\omega) < 0.$$

Finally consider $\varphi = 1$. We re-display the three relevant rows for convenience:

state	R	R
ss	$r(ss \rightarrow s) - r(s \rightarrow ss) + r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$\Pi_h(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi) + R(s)$
s	$r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U - \Pi_h(\eta)((1 - \omega)\tau + \omega(1 - \tau)) - u((1 - \omega)\tau + \omega(1 - \tau), 1, \varphi) + \Pi_b(\eta) + u(1, 1 - \tau, \varphi)$
ww	$r(ww \rightarrow w) - r(w \rightarrow ww) + r(w \rightarrow sw)$	$U + \Pi_h(\eta)((1 - \omega)(1 - \tau) + \omega\tau) + u((1 - \omega)(1 - \tau) + \omega\tau, 0, \varphi) - \Pi_h(\eta)\omega - u(\omega, 0, \varphi)$

To compute $R(ss) - R(ww)$ observe that the U terms and all the $\Pi_\zeta(\eta)$ terms cancel out. Hence

$$\begin{aligned} R(ss) - R(ww) &= u(1 - \omega, 1, \varphi) - u(1, 1, \varphi) - u((1 - \omega)\tau + \omega(1 - \tau), 1, \varphi) + u(1, 1 - \tau, \varphi) \\ &\quad - u((1 - \omega)(1 - \tau) + \omega\tau, 0, \varphi) + u(\omega, 0, \varphi) \end{aligned}$$

We have $u(y_a, y_d, \varphi) = \alpha [-y_a + (1/2)y_a^2]$. All the linear terms cancel out, so we compute all the quadratic terms, that is the sum of squared offensive elements S_a^2 . We get

$$\begin{aligned} S_a^2 &= -1 + (1 - \omega)^2 - ((1 - \omega)\tau + \omega(1 - \tau))^2 + 1 - ((1 - \omega)(1 - \tau) + \omega\tau)^2 + \omega^2 \\ &= 2\tau(1 - \tau)(1 - 2\omega)^2 \end{aligned}$$

which is positive since $\omega < 1/2$. □